DESUBLIMATION OF FLOWING VAPOR WITH A THERMAL RESISTANCE BETWEEN THE COOLANT AND THE HEAT SINK SURFACE

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Results are shown of a study concerning the desublimation of water vapor flowing between parallel plates, with a finite coefficient of heat transfer between coolant and heat sink surface.

In [1] the authors have presented results of a study concerning the desublimation of water vapor during its flow between parallel plates. The surface temperature was there assumed to remain constant.

In practice, however, the desublimation process occurs often under conditions where the heat of phase transformation is carried away along a path with a significant thermal resistance. This is the case, for instance, in a desublimator with the heat sink surface cooled by a slowly circulating coolant.

It is of interest, therefore, to analyze the laws which govern the desublimation of water vapor when the complicating effect of thermal resistance is taken into account.

The theoretical analysis of this process will be based here on the general concepts about the phenomenon and on the assumptions made earlier in [1]. It will also be assumed that the heat transfer coefficient is independent of the heat load. The system of equations describing the desublimation process will be written as:

$$\frac{\partial h}{\partial \tau} = \frac{2a\beta^2}{h+\Delta}, \tag{1}$$

$$\int\limits_0^{X(\tau)} h(\tau, x)\,dx = \frac{j\tau}{\rho b} \tag{2}$$

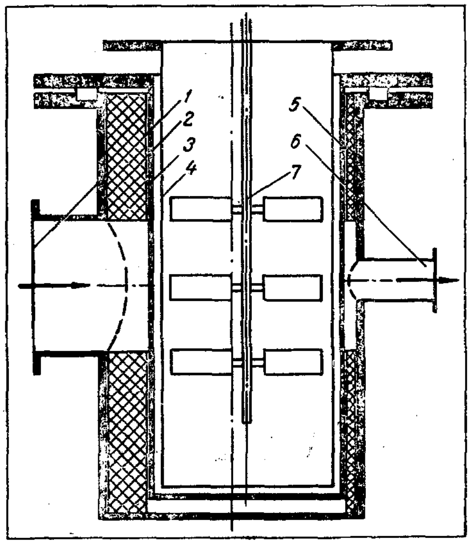

Fig. 1. Schematic diagram of the test segment: 1) inlet tube for the vapor–gas mixture; 2) case; 3) metal cup; 4) glass cup; 5) insulation; 6) air drain tube; 7) mixer.

with the boundary conditions

$$h = 0,\ x = 0 \quad \text{at} \quad \tau = 0, \tag{3}$$

$$h = 0,\ x = X(\tau) \quad \text{at} \quad \tau > 0, \tag{4}$$

where $\Delta = \lambda(1/\alpha + \sum\limits_{i=1}^{n} R_i)$; $\beta^2 = c(T_s - T_f)/2r$.

An exact solution to system (1), (2) cannot be found. In order to find an approximate solution, we will proceed from the following premise.

As is well known [2], the desublimation rate becomes always finite when thermal resistances exist between the heat sink surface and the coolant ($\Delta > 0$). When the vapor flows along a plate, therefore, then the length of the desublimation surface becomes finite too and the process starts at a definite instant of time. We note that, when $\Delta = 0$, the initial desublimation rate (when no desublimate has been formed yet) is infinite while the length of the desublimation surface is zero.

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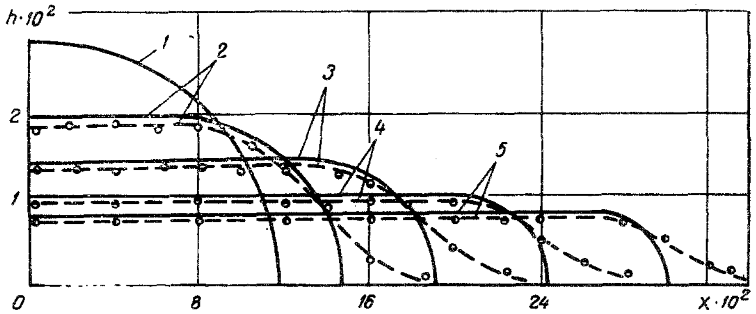

Fig. 2. Ice profile along a plate (P = 333 N/m²; T_f = 208°K; τ = 7200 sec; b = 65·10⁻³ m; j = 21·10⁻⁶ kg/sec; j_A = 5·10⁻⁹ kg/sec): Δ = 0 (1), 0.010 m (2); 0.02 m (3); 0.03 m (4); 0.04 m (5). Solid lines represent calculated values, dashed lines represent test values.

Thus, if initially the vapor desublimates on a finite area, then the height of the interphase boundary is obviously a function of time only. Therefore, Eq. (2) may be rewritten as

$$\int_0^{X_0} h_0 \ \tau \ dx + \int_{X_0}^{X(\tau)} h(\tau, \ x)\, dx = \frac{j\tau}{\rho b} . \tag{5}$$

Here the first integral represents the volume of ice on the initial area. Function $h_0(\tau)$ under this integral is determined from (1) easily, inasmuch as this is a function of time τ only:

$$h_0(\tau) = \sqrt{4a\beta^2\tau + \Delta^2} - \Delta. \tag{6}$$

The size of the initial area (X_0) will be determined from the equality

$$\frac{j}{\rho b}\delta\tau = X\delta h. \tag{7}$$

which is valid for a short time interval $\delta\tau$ measured from the start of the process.

Having expanded h into a Taylor series and retaining the first two terms only, we obtain for $\delta\tau \to 0$

$$X_0 = \frac{j\Delta}{2b\rho a\beta^2} . \tag{8}$$

Consequently, the first integral in Eq. (5) may be written as $\pi\Delta h_0/4\beta k$ where $k = \pi\rho a b\beta/2j$.

In order to evaluate the second integral, we assume the ice distribution over the distance $X \geq X_0$ to correspond to an ice distribution with boundary conditions of the first kind [1], i.e., we represent it by a quadrant of an ellipse with one semiaxis h_0 and the other semiaxis found according to Eq. (5):

$$\frac{j\tau}{\rho b} - h_0 X_0 = \frac{\pi}{4} h_0 (X - X_0), \tag{9}$$

wherefrom

$$(X - X_0) = \frac{h_0}{2\beta k} .$$

The total length of the desublimation zone is

$$X = \frac{2h_0 + \pi\Delta}{4\beta k} . \tag{10}$$

The ice distribution on the interval $X_0 \leq x \leq X$ is determined from the equation of the ellipse:

$$h = \sqrt{h_0^2 - 4\beta^2 k^2 \left(x - \frac{\pi\Delta}{4\beta k}\right)^2} . \tag{11}$$

This approximate solution satisfies the boundary conditions (3), (4) and the integral Eq. (5), and on the interval $0 \leq x \leq X_0$ the differential Eq. (1).

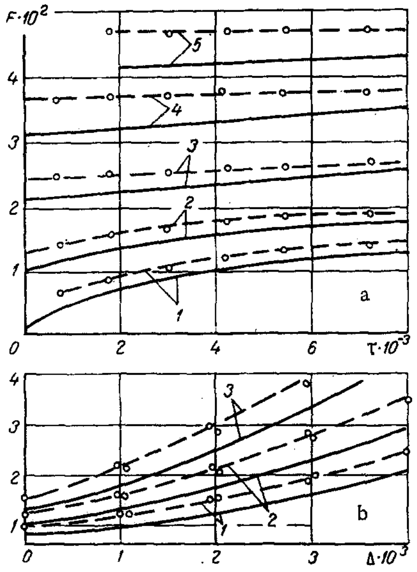

Fig. 3

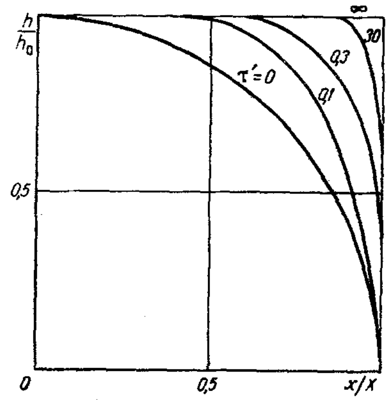

Fig. 4

Fig. 3. Surface F (m^2) occupied by ice during desublimation of water vapor on a plate (T_f = 208°K; j = $21 \cdot 10^{-6}$ kg/sec, $j_A = 5 \cdot 10^{-9}$ kg/sec): (a) as a function of time at a constant pressure P = 66.6 N/m^2 and various fictitious layer thicknesses Δ = 0 (1), 0.010 m (2), 0.020 m (3), 0.030 m (4), 0.040 m (5); (b) as a function of the fictitious layer thickness Δ (m) at a fixed process time τ = 7200 sec and various pressures P = 333 N/m^2 (1), 66.6 N/m^2 (2), 13.3 N/m^2 (3). Solid lines represent calculated values, dashed lines represent test values.

Fig. 4. Dimensionless profile of desublimate at various values of τ'.

The validity of this solution was verified experimentally with the apparatus described in [1]. The test segment for this was, however, novel in principle (Fig. 1).

The vapor-gas mixture entered through the inlet tube 1, then flowed through the channel between cylindrical cups 2 (outer) and 3 (inner) with a foam-plastic insulation 5. The vapor desublimated on the outside surface of cup 3. In order to generate the required thermal resistance between coolant and heat sink surface, a glass cup 4 of a specific diameter was placed into cup 3. The gap between them was filled with a liquid of known thermophysical properties (methyl or ethyl alcohol, acetone, or similar). The glass cup was filled with a mixture of carbon dioxide and alcohol rapidly mixed with a stirrer 7. Inasmuch as the diameter of the glass cup 3 (200 × 5 mm) was much larger than the maximum thickness of the desublimate layer (10-15 mm), the effects of curvature of the heat sink surface could be disregarded.

The tests were performed with the coolant at a temperature of 208°K and the vapor-gas pressure varying from 10 to 600 N/m^2. The fictitious thickness of the ice layer (Δ) changed from 1 to 40 mm, corresponding to a change in the heat transfer coefficient from 2000 to 50 W/m^2·°C. After each test, the test segment was disassembled and the ice thickness, base area, and total weight were measured.

For illustration, the theoretical ice profile is compared in Fig. 2 with the results of measurements. It is evident here that within the end zones the test data deviate appreciably from the theoretical ones. This is due to the presence of air in the system, i.e., the desublimation rate within an end zone is determined by the rate of vapor diffusion toward the heat sink surface. The relative size of the surface occupied by ice in this zone decreases with time and as the velocity of the vapor stream increases (Fig. 3).

In conclusion, it is quite worthwhile to analyze some of the results obtained by theoretical calculations.

A family of curves representing the desublimate profile along a plate is shown in Fig. 4 in dimensionless coordinates, for various values of the parameter $\tau' = \tau_\Delta/\tau$. Here τ_Δ is defined as the buildup time of the fictitious ice layer thickness (Δ), according to the formula

$$\tau_\Delta = \frac{\Delta^2}{4a\beta^2} . \qquad (12)$$

The graph indicates that at $\tau' \to 0$ the ice profile approaches a circle, as in [1], while at $\tau' \to \infty$ it acquires the shape of a square. This observation may be applied to the design of industrial desublimators. Namely, the entire design may be based on the assumption of a rectangular ice profile when the heat transfer coefficient is low. When the heat transfer coefficient is high, as in the case of immediate boiling of the coolant in channels in the heat sink surface, then the design may be based on formulas (10) and (11) with the assumption that $\Delta = 0$. In both cases the error in calculating the surface will not exceed 22%.

NOTATION

h is the thickness of ice layer;
τ is the time;
a is the thermal diffusivity;
x is the length coordinate;
X is the length of desublimation zone;
j is the mass flow rate of vapor;
j_A is the mass flow rate of air;
ρ is the density of ice;
b is the width of plate;
λ is the thermal conductivity;
c is the specific heat of ice;
r is the heat of phase transformation;
R is the thermal resistance;
α is the heat transfer coefficient;
T_s is the saturation temperature;
T_f is the temperature of cooling fluid;
X_0 is the initial length of desublimation zone.

LITERATURE CITED

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